(1) We’re interested in the units in a ring $R$ for several reasons having to do with the multiplicative structure of $R$. For instance, if $u$ is a unit, we can always solve an equation of the form $ux = r$, for any $r \in R$. And units always give us “inessential” or “trivial” differences in factorizations: if $u$ is a unit, we have trivial factorizations like $r = u(u^{-1}r)$ and if $r = ab$ is a factorization, we also have factorizations like $r = (au)(u^{-1}b)$. Even when we talk about this in $\mathbb{Z}$, where the only units are 1 and $-1$, we avoid the issue of units in school mathematics by defining primes to be whole numbers greater than 1 and only stating the Fundamental Theorem of Arithmetic for positive integers. But in most rings, we don’t have such an easy separation into positive and negative parts that lets us hide the issue of units.

(2) Section 7.2 talks about a special kind of unit, one that has a power equal to 1. These are called roots of unity (sometimes roots of 1). In $\mathbb{Q}$ or $\mathbb{R}$, the only roots of unity are $\pm 1$; in $\mathbb{C}$, there are $n$ distinct $n$-th roots of unity for each positive integer $n$, but most units are not roots of unity (all the nonzero elements of $\mathbb{C}$ are units). But Proposition 7.6 shows that if there are only finitely many units in $R$, all of them are roots of unity. This means that if $R$ itself is finite, all the units are roots of unity and we can study the units by studying the roots of the equations of the form $x^n = 1$ for various $n$. The theorems of Fermat and Euler that the textbook discusses have to do with the units in rings of the form $\mathbb{Z}_m$ ($m$ prime for the Fermat theorem).

(3) Section 7.4 talks about how to compute the Euler $\varphi$ function, which gives the number of positive integers less than or equal to $m$ that are relatively prime to $m$ (and hence the number of units in $\mathbb{Z}_m$). This is presented as a combinatorial problem and the textbook uses the combinatorial principle of inclusion-exclusion to solve it. I won’t be going into this in class, but $\varphi(m)$ is an interesting thing to talk about in high school math, and you will see (or have seen) more about the principle of inclusion-exclusion in Math 455.

(4) As I said in the assignment, I won’t be talking about Section 7.5 on the RSA public-key encryption scheme in class, but this is an interesting application of some elementary number theory and algebra that has practical applications. So it’s a good thing to have in your bag of extra topics/projects/etc. for high school students.