Math 490A—Fall 2019
Homework due September 19

Base-10 (or, more generally, base-\(b\)), representation of integers

Almost all of school mathematics (including all the algorithms for arithmetic, etc.) relies on our base-10 system for representing numbers; understanding and being fluent in working with this system is a key part of school mathematics. (This system is also related in important ways to the ways we think about polynomials, as we’ll see later.) But how do you know that you can represent any integer this way? Prove the following proposition. (Hint: Use the division theorem and induction. You might also want to think about the fact that there is some \(\ell \in \mathbb{N}\) such that \(b^{\ell+1} > m \geq b^\ell\).)

**Proposition.** If \(b \geq 2\) is an integer, then every positive integer \(m\) has an expression “in base \(b\)”: there are integers \(d_i\) with \(0 \leq d_i < b\) for all \(i\) and \(d_k \neq 0\) such that

\[
m = d_k b^k + d_{k-1} b^{k-1} + \cdots + d_1 b + d_0.
\]

Moreover, this is expression is unique.

The numbers \(d_k, d_{k-1}, \ldots, d_0\) are called the \(b\)-adic digits of \(m\).

Do the following problems from the textbook

- Exercise 6.7
- Exercise 6.8
- Exercise 6.10
- Exercise 7.3
- Exercise 7.9