Give precise and complete mathematical definitions of the following.

1. (10 points) A graph is . . .

**Solution:** a finite set $V$ together with a set of 2-element subsets of $V$. (Note that “2-element subsets” are not the same as unordered pairs: the pair $(v, v)$ counts as an unordered pair, but $\{v, v\}$ is not a 2-element subset.)

2. (10 points) The degree of a vertex $v$ in a graph $G$ is . . .

**Solution:** the number of edges of $G$ that contain $v$. (This is “contain” in the sense that $v$ is an element of the edge as a 2-element subset of the vertex set. You could also say “the number of edges incident with $v$”.)

3. (10 points) A trail in a graph $G$ is . . .

**Solution:** a sequence $v_1, \ldots, v_k$ of vertices of $G$ such that $v_iv_{i+1}$ is an edge in $G$ for each $i = 1, \ldots, k - 1$ and there are no indices $i \neq j$ such that $v_i = v_j$ and $v_{i+1} = v_{j+1}$, that is, no edges repeat. (On this quiz, I accepted something like “a walk in which no edges repeat”, but in the future, when we defined things more or less together as we did with walk and trail, I will expect you to give the more complete definition. Or at least to say a trail is a walk in which no edges repeat and then give a definition of walk. But it’s always ok to use terms defined earlier in the course when giving a definition on a quiz.)