Math 300.3—Spring 2019
Review Sheet for Exam 1

The exam will be given in class on Thursday, February 28. If, due to an emergency, you are unable to take the exam then, it is your responsibility to notify me at the earliest possible time.

You should know the axioms, definitions, and results from the textbook and class, and be able to use them in answering questions and doing proofs. I may ask you to state definitions, theorems, etc., and to prove things. I won’t ask you to give proofs of theorems proved in the textbook, but you should be able to use the techniques used in those proofs. The homework is a good general guide to the kind of problems you’ll see, though some of the homework problems are probably too long for an in-class exam. So make sure you have gone through the posted solutions to homework problems.

What do I mean by “you should know” things like definitions and results? I don’t mean that you should necessarily be able to refer to Exercise 32 or Definition 17 by number. And you don’t need to be able to quote things with exactly the same wording used in the book. But you need to know what the definitions, etc., actually say and you need to know it precisely and completely in the sense of the mathematical content.

For example, the book says

Let $n \in \mathbb{N}$. If $a$ and $b$ are integers, then we say that $a$ is congruent to $b$ modulo $n$ provided that $n$ divides $a - b$.

If the test asks you for the definition of “congruence modulo $n$”, you don’t need to quote this word for word, using the same letters, etc. But you would need to give a definition that is mathematically equivalent to the one in the book. So it would be fine to say something like

Two integers are congruent modulo $n$ if their difference is divisible by $n$.

But saying it means $a \equiv b \pmod{n}$ won’t get you any credit.

For proofs, you can use results from the book, homework problems, or discussion in class, but you need to cite those results. You don’t need to cite theorems by the number in the book, but you should say “A theorem in the book says that . . . ” or something like that which makes it clear exactly what result your referring to.

You’ll have the whole class period (but people who come late won’t get extra time without a very good reason). No notes, books, or electronic devices can be used during the exam.

The exam covers the material in the text through Section 4.2. You’re responsible for the material in the textbook and everything we’ve done in class. Here’s a bit more detail on the topics (though the fact that something is listed here doesn’t mean it will necessarily be on the exam and the fact that something isn’t mentioned here doesn’t mean it won’t be on the exam).
Chapter 1: Section 1.1 covers the basic ideas of statements, implication, and introduces truth tables. It also notes some of the basic closure properties of the integers, rationals, etc., that we use explicitly in proofs. Section 1.2 introduces simple direct proofs. All of these ideas are developed more thoroughly in the later chapters, so I’ll comment more specifically on what you need to know when I discuss those.

Chapter 2: Section 2.1 introduces the logical operators negation, conjunction, and disjunction, and deals with implication in more detail, including wording and the biconditional operator. Truth tables are explored more thoroughly here. Given truth values for the elementary propositions in a complex compound statement, you should be able to determine the truth values for the compound statement. One way to do this is to construct a full truth table—you should expect at least one question asking you for the truth table for a compound statement.

Section 2.2 introduces the idea of logical equivalence and discusses equivalent forms of the standard implication $P \rightarrow Q$, as well as the negations of an implication and other compound statements. You need to be able to work with the negation of disjunction and conjunction (De Morgan’s Laws).

After setting up some set theory, including the idea of universal set for some discussion and the empty set (this is mostly as notation; we’ll do more on set theory in Chapter 5), Section 2.3 looks at open statements, or predicates. A key idea here is the truth set of a predicate (in terms of a given universal set). Section 2.4 focuses on the universal and existential quantifiers, which can be used to turn an open sentence into statements about the truth set of an open sentence. You should be able to work with quantifiers, both in English and in logical notation, and understand the negations of quantified statements.

Chapter 3: Chapter 3 is focused on some proof methods (and the writing styles that are used with them), including methods for direct proofs (including proving a logically equivalent statement), proof by contradiction, and case-by-case arguments. (Remember that in proving a theorem by considering separate cases, it’s important to make sure that you cover all the possibilities!). These are all methods that you should be able to use; the summary in Section 3.6 and the notes on the course web page are guides about when particular methods might be appropriate, but there’s no recipe. If you try one method and aren’t getting anywhere, see if there’s a way to apply another method.

Section 3.5 introduces the division algorithm and the “congruent modulo $n$” relation. They’re in this chapter mostly to provide rich examples for the application of various proof methods. We proved the division algorithm in class, relying on the Well-Ordering Principle that states that any nonempty set of natural numbers (or whole numbers) has a least element. This, as we discussed in class, is equivalent to the Principle(s) of Mathematical Induction in Chapter 4. I won’t ask you to reproduce the proof of the division “algorithm”, but you should know what it says and you should be able to work with congruences if I
Chapter 4: You need to know Sections 4.1 and 4.2 for the exam; we haven’t covered 4.3 yet.

Section 4.1 covers standard induction, used to prove a statement of the form “For every natural number \( n \), \( P(n) \) is true.” Section 4.2 extends this to handle statements of the form “For every integer \( z \geq M \), \( P(z) \) is true” and then introduces the Second Principle of Mathematical Induction (also called “strong induction” or “complete induction”). You should be able to use induction to prove things. We used this Second Principle to handle things, like the proof that every natural number is a product of primes, where we prove \( P(k+1) \) using \( P(\ell) \) for one or more values of \( \ell < k+1 \), not necessarily \( k \). If you’re doing an induction proof on the exam, be careful to state the base case and the induction step fully, distinguishing between what you’re assuming in the induction step and what you’re trying to prove. Make sure that your argument for the induction step works for all \( k \); remember what goes wrong with the “proof” that all dogs are the same breed.