In these problems, you will construct the integers from the set of whole numbers, \( \mathbb{W} \), in a way similar to that by which we constructed \( \mathbb{Q} \) from \( \mathbb{Z} \) in class. (We can construct \( \mathbb{W} \) from set theory, so this gets us from set theory to \( \mathbb{Q} \).)

2. Let \( X = \mathbb{W} \times \mathbb{W} \). Define a relation \( \sim \) on \( X \) by

\[(m, n) \sim (p, q) \text{ if and only if } m + q = p + n.\]

Show that \( \sim \) is an equivalence relation.

3. Let \( \mathbb{Z} \) be the set of equivalence classes, \( X/\sim \). Show that the function \( i: \mathbb{W} \rightarrow \mathbb{Z} \) given by \( i(w) = [(w, 0)] \) is one-to-one.

4. Define addition on \( \mathbb{Z} \) by

\[((m, n)] + [(p, q)] = [(m + p, n + q)].\]

(a) Show that this is well-defined.
(b) Show that \( i(w) + i(v) = i(w + v) \) for all \( w, v \in \mathbb{W} \).
(c) Show that \( i(0) \) is the unique identity element for addition in \( \mathbb{Z} \).
(d) Show that addition in \( \mathbb{Z} \) is commutative.

5. Define multiplication on \( \mathbb{Z} \) by

\[((m, n)] \cdot [(p, q)] = [(mp + nq, np + mq)].\]

(a) Show that this is well-defined.
(b) Show that \( i(w) \cdot i(v) = i(wv) \) for all \( w, v \in \mathbb{W} \).
(c) Show that \( i(1) \) is the unique identity element for multiplication in \( \mathbb{Z} \).
(d) Show that the distributive law holds for multiplication and addition in \( \mathbb{Z} \).