Section 6.1

1. For each of the following word problems, write the corresponding numerical division problem, state which interpretation of division is involved, and solve the problem

   a. If 252 rolls are to be put in packages of 12, then how many packages of rolls can be made?
      
      \[ 252 \div 12 = 21 \]
      
      This is the “how many groups” interpretation; a group is a package of 12 rolls.

   b. If you have 506 stickers to give out equally to 23 children, then how many stickers will each child get?
      
      \[ 506 \div 23 = 22 \]
      
      This is the “how many in each group” interpretation; a group is the stickers that a particular child gets.

   c. Given that a gallon is 8 pints, how many gallons of water are 48 pints of water?
      
      \[ 48 \div 8 = 6 \]
      
      This is the “how many groups” interpretation; a group is 8 pints.

   d. If your car used 12 gallons of gasoline to drive 360 miles, then how many miles per gallon did your car get?
      
      \[ 360 \div 12 = 30 \]
      
      This is the “how many in each group” interpretation; a group is the number of miles driven on one gallon of gasoline.

   e. If you drove 177 miles at a constant speed and if it took you 3 hours, how fast were you going?
      
      \[ 177 \div 3 = 59 \]
      
      This is the “how many in each group” interpretation; a group is the number of miles driven in an hour.

   f. Given that 1 foot is 12 inches, how many feet long is an 84-inch-long board?
      
      \[ 84 \div 12 = 7 \]
      
      This is the “how many groups” interpretation; a group is 12 inches.

2. Write two word problems for \( 63 \div 7 \), one for each of the two interpretations of division. Solve each problem.
There are 63 textbooks to shelve. The student shelving books can carry 7 textbooks at a time. How many trips does it take to shelve the books? This is a “how many groups” problem, where a group is the set of 7 textbooks that the shelver can carry at one time. The answer is 9.

There are 63 students in a class doing group project. If there are 7 groups, how many students are in each group? This is a “how many in each group” problem. The answer is still 9, of course.

3. a. Write an array problem for $21 \div 3 = ?$ and make a math drawing for the problem.

   Obviously there can be a lot of different answers for this problem. Here’s one: A UMass classroom has 21 tables with 3 tables in each row. How many rows are there?

   A math drawing would be something like this:

   ☐ ☐ ☐  ☐ ☐ ☐  ☐ ☐ ☐  ☐ ☐ ☐  ☐ ☐ ☐  ☐ ☐ ☐  ☐ ☐ ☐  ☐ ☐ ☐  ☐ ☐ ☐  ☐ ☐ ☐

   b. Write an area problem for $21 \div 3 = ?$ and make a math drawing for the problem.

   A friend has a rug that she is willing to give you for free. She knows it is 3 feet wide and covered 21 square feet. How long is it?

   A math drawing would be something like this:
7. Write and solve one word problem for \(35 \div 70\) and another for \(70 \div 35 = ?\). Say which is which.

We have 35 pounds of fertilizer to give to 70 gardeners. How much fertilizer does each gardener get? \((35 \div 70)\)

A punch recipe uses 70 cups of orange juice. If 35 people share the punch equally, how much orange juice does each person drink? \((70 \div 35)\)

Section 6.2

1. Use the definition of fractions and how-man-units-in-1-group division to explain in your own words why \(3 \div 7 = \frac{3}{7}\). Your explanation should be general, in the sense that you could see why \(3 \div 7 = \frac{3}{7}\) would still be true if other numbers were to replace 3 and 7.

If there are 3 identical pies to be divided equally among 7 people, the how-many-in-1-group interpretation says that each person will get \(3 \div 7\) pies. So we want to see that, in terms of fractions, each person will get \(\frac{3}{7}\) of a pie, meaning that \(3 \div 7\) is the same as \(\frac{3}{7}\).

Here’s how we can see how much pie each person gets as a fraction of a pie. One way to divide the pies equally is to divide each pie into 7 equal parts (so each part is \(\frac{1}{7}\) of a pie, and give each person one part from each of the 3 pies. This clearly means that each person gets \(\frac{3}{7}\) of a pie.

Charge model problems

a. \(-3\) multiplied by \(-4\)

We want to start with no charge and take away 3 groups (because that’s what multiplying by -3 means) of 4 negative charges. So we have to start with a representation of 0 charge that has 3 groups of 4 negative charges that we can take away. Here’s one.
Now we can take away the 3 groups 4 negative charges, leaving

\[ \circ \bullet \circ \bullet \circ \bullet \circ \bullet \circ \bullet \circ \bullet \circ \bullet \circ \bullet \]

We see that this is a representation of 12 (since it has 12 positive charges and no negative charges).

b. \(-4\) divided by \(-2\)

We first interpret this as multiplication with a missing factor. Depending on whether we use the how-many-groups interpretation of the how-many-in-one-group interpretation, we would think of the division problem as \(? \times (-2) = -4\) or \((-2) \times ? = -4\), respectively. The “?” is our quotient.

In the how-many-groups version, we are asking how many groups of 2 negative charges do we have to add or take away from nothing to get 4 negative charges. It’s clear that we could add two groups of 2 negative charges to get 4 negative charges, so we have \(2 \times (-2) = -4\), which is the same as \((-4) \div (-2) = 2\).

In the how-many-in-one-group version, we are saying we will take away 2 groups of charges (multiplying by \(-2\)) to be left with 4 negative charges, and we want to know the charge of each group. We know that a representation of zero charge has the same number of positive charges and negative charges, and taking away positive charges will leave us with an excess of negative charges. We want to end up with 4 more negative charges than positive charges, representing \(-4\), after taking away 2 groups. So we want 2 positive charges in each of the 2 groups we take away: \((-2) \times 2 = -4\).